

THE  
THEORY  
OF  
HARMONICS:  
OR,  
AN ILLUSTRATION  
OF THE  
GRECIAN HARMONICA.  
IN TWO PARTS:

- I. As it is maintained by EUCLID, ARISTOXENUS, and  
BACCHIUS SENIOR.
- II. As it is established on the Doctrine of the RATIO: in which  
are explained the Two DIAGRAMS of GAUDENTIUS, and  
the PYTHAGOREAN Numbers in NICOMACHUS.

With PLATES, an INTRODUCTION to each PART, and a  
General INDEX.

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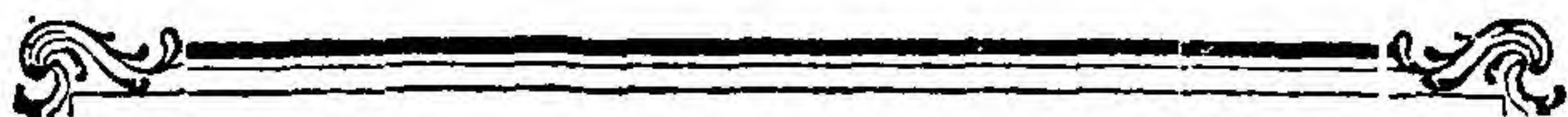
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## INTRODUCTION.

**I** HAVE no other excuse for troubling the Public on a subject that has already been treated of by so many learned writers, than the certainty of having made some discoveries, which may, in part, remove the great obscurity that has so long attended the most curious and diligent enquiries after the true principles of the ancient Grecian Harmonica. The advantages I have received in my profession from that doctrine, would never have been a sufficient temptation to appear as a writer, particularly on a subject so difficult to be explained, so little qualified as I am to do justice to it, were I not in hopes that the knowledge of it may be very useful to professors, by the great advantages they may receive from the true and unerring principles of

B Science.



Science. To explain the ancient doctrine of the Harmonica, and the several parts into which it is divided, and to reconcile it to modern theories, is the great object that has engaged me in this pursuit; and however conscious I am of my own inabilities, yet truth, divested of ornament, cannot fail of meeting with indulgence from the Public, especially when it leads to useful discoveries.

Nothing contributes so much to the encouragement of Study, as the knowledge of some governing and leading principle; some visible and faithful guide, that will conduct us through the mazes of Science, and teach us to love and obey her laws. It is this that warms and animates our endeavours in the arduous pursuit, and in the end rewards our labours with success.

This governing principle shews itself in no part of human learning so much as in the various operations of numbers; whose powers, by a kind of magic, have greatly contributed to the many discoveries and improvements that have been made in all arts and sciences; nor can it be otherwise,  
while



while Truth is the great object to which those powers are directed.

Where the variety of proportion and its different relations constitute the beauty and harmony of parts, from which the great perfection of any art or science is derived, the application of some branch of the Mathematics is indispensibly necessary, since without its friendly aid we must wander in obscurity and uncertainty.

From this great source the first principles of Music were discovered ; and from the same fountain others must be drawn, if we would be led by the most unerring guide towards perfection, to which point the efforts of the professor, however weak, are directed.

The usual method of treating this subject has been such, that without a profound knowledge of the Mathematics, together with the finer and more sublime parts of Geometry, he is excluded from the advantage which the writings of so many learned authors would give him, and is left to the uncertain



rules and examples of his practical predecessors, with what nature, and his own feelings, will dictate.

In this situation there remains no hope but that of uniting the practical with the demonstrative, the art with the science ; at least as far as a perfect knowledge of Harmonics is concerned ; leaving the more philosophical parts to others.

In order to effect this, the operations of numbers must be the most simple, and their application to sounds the most rational and easy to be comprehended,

This ease and simplicity will naturally engage the attention of the professor, who will readily give up some part of his time from that more severe and laborious practice, which all instruments now require, to a study that will not only lessen his labours, but at the same time increase his mechanical powers, and raise his reputation on the most solid and lasting foundation.

Among



Among the theories which have appeared at different periods, those of the Greeks seem to have all the advantages that can be wished to lead us to the true knowledge of Harmonics ; for as their principles are in nature, they must be fixed and immutable.

This appears in no part so obvious and certain, as in the formation of their Scales, which, from the impossibility of altering their original order, must ever remain in the same form and proportion in which they have always been ; as the different combinations of Time are naturally confined to the two Primes, 2 and 3, or their Multiples, from which there can be no variation.

Hence we may conclude, it is scarcely possible that this natural and absolute necessity in the order and disposition of Sounds should have long escaped the observation of the ancients ; or that they should not soon have discovered the principles by which Tune was so limited and circumscribed ; especially as their theories were not formed by mere practical musicians, but by men of learning, who taught  
them



them publicly in their schools, together with other arts and sciences, proper for the most finished education.

But that which confirms this opinion of their knowledge in the Theory of Harmonics, is the simplicity of the Tetrachord, which always continued their favourite system. For however they disagreed with respect to the necessity of measuring intervals by the Ratio, yet they were all unanimous in asserting the perfection of the Tetrachord, which is evident from its making so essential a part in all their Diagrams.

That a system, so simple as this, should continue for so many ages without the least alteration, is not only a proof of the perfection of its composition, but that it has its foundation in nature ; or in such laws as could not be altered without destroying the original harmonious constitution of the system, and, with it, the order and regularity of the scales.

By



By what means it was laid aside in the tenth century, and succeeded by Guido Aretine's Hexachords, ( the next ancient system ) will be observed in page 13.

As to the particular time when the Tetrachord was first used, or by whom invented, it is not very material to enquire, since there can be no doubt of its being the most ancient musical system in the world, And if we would wish to know more with respect to the great antiquity of Music, the sacred and prophane writers will abundantly inform us : for as it was very early used in religious and other ceremonies, its improvement, in an equal degree with other arts and sciences, seems a natural consequence.

Aristoxenus gives it the highest antiquity ; where in speaking of the three genders, the Diatonic, Chromatic, and Enharmonic, page 19, he says, “ The Diatonic is to be placed as first and  
“ most ancient, and it is that which human nature  
“ first receives.”

But



But without insisting on this, it must be granted, that the human voice is tuned by this scale or gender, which is universally the same, at all times, and in all countries; for as this is the scale of nature, there is sufficient reason to believe, that the principles of harmony, the number and quality of consonances and dissonances, are fixed and determined by certain laws, which, therefore must remain unalterable, whether they are understood in part, or in the whole. Thus, the blood circulated before the great Harvey was born, and the laws of gravity and attraction operated, before that great ornament of this country, Sir Isaac Newton, existed; nor could their discoveries make any alteration in those laws of nature, for they were constantly and uniformly the same, both before, as well as after.

The many wants to which the condition of man is subject, contribute by a kind of necessity to the improvement of all arts and sciences, except music; but in this, where pleasure is the end proposed, we are too impatient to be gratified, and the charm is too powerful to admit of delay from the cold and  
dry



dry investigation of science. This is perhaps one reason why we meet with so few theories.

Notwithstanding the great antiquity of Music, we have no writers come down to us prior to the Grecians; and, of these, the number is not very considerable.

In them there is mention made of earlier writers, of whom a large catalogue may be seen in the 3d book of Fabricius's *Bibliotheca Græca*.

The great and wonderful effects which the Grecian musicians are said to have had over the passions, have inclined the moderns to believe that the art has long since been lost, as we meet with no such effects now; and therefore they conclude we are entirely ignorant of the noble and sublimer parts, which operated with such an irresistible force.

Some have taken the contrary side, endeavouring to prove those extraordinary and wonderful effects to be mere fable, and the invention of the poets. Others have denied that the Grecians had

C

any



any knowledge of harmony, allowing them melody only, and limiting their Chorusses to unison or octave ; by which they have put their music on a level with common ballad-singing.

This opinion is principally founded on their Scales or Diagrams, which, if they were such as have hitherto been explained, must extinguish every sense of musical feeling, by imposing a series of such inconcinuous sounds, as it is impossible for the voice to move in, or the ear to approve. The truth of this may be made very evident from tuning a harpsichord by perfect fifths throughout, by which the several degrees will be tones major and limmas. From this experiment it will be found, that not only harmony, but even a perfect melody, cannot be obtained from principles so opposite to nature.

These writers, in favour of the moderns, have not been aware of this very great objection ; they did not consider, that if the scale contained the true and natural degrees of voice proper for melody, (which they acknowledge the Greeks had improved to the greatest perfection) it must also have been  
proper



proper for harmony ; which, agreeably to the laws of science, is rather the Parent than the Offspring of melody, as will appear an undeniable fact in the course of this work.

If however the true Diatonic, which is the scale, or voice of nature, has not yet been found in their diagrams, it is not impossible, but they may admit of some other construction, some other interpretation, which may lead us to a more certain knowledge of their doctrine, and the true principles by which their Harmonica is to be understood.

But before I proceed, it may be necessary to take a flight view of the theories formed by the Latins, in order to recover music from the obscurity in which it lay buried for many ages.

This enquiry will be the more necessary, as it will lead us to a better knowledge of the Grecian theories, and enable us, in some measure, to judge how far the Latins deserve the honour of being not only the restorers of music, but also the inventors of harmony ; or, which is the same, of uniting



two, three, or more different melodies, agreeably to the laws of harmony : a discovery of the greatest importance, and which by many is said to have been unknown to the Greeks.

Musick must have been in a very weak and infant state among the Latins, if we may judge by their systems ; from which it appears, that the few tracts of the Grecians in their possession were of little service to them. For, in these tracts, the musical characters were numerous, and a great variety of distinctions was necessary to their right application. These, with many other difficulties, were not easily got over ; and it is not to be wondered, that the Latins chose to substitute a part of their own alphabet, rather than continue so great a variety of characters, as was used by the Greeks : for as the compass of their system was the same, it required only their first fifteen letters, from A to P, to be applied from grave to acute to compleat the Bifdiapason.

Hence it appears, that the Grecian doctrine must have been very ill understood, if a system of so little meaning could be adopted, whose great imper-  
fections



fections required alteration. Accordingly, Pope Gregory, some time after, discovered that the whole was no more than a conjunction of two octaves, and that the return of the tones and semi-tones in the acute, was in the same position as in the grave; and therefore he reduced the whole system to a Heptachord, (a system of seven sounds) to which he applied the first seven letters of the alphabet, in the following order, to distinguish the Grave, the Mean, and the Acute.

A.B.C.D.E.F.G. || a.b.c.d.e.f.g. || aa.bb.cc.dd.ee.ff.gg.

This Gregorian scale continued without any material alteration till about the eleventh century; when Guido Aretine, a Benedictine Monk, formed a new system composed of three Hexachords, each consisting of six sounds, which he called by the names of the B quadro Hexachord, the natural Hexachord, and the B flat, or moll, Hexachord.

The gravest sound of the B quadro was our G, which is a tone lower than the lowest in the Grecian Diagram, and which he called for that reason by the  
name



name of Hypoproslambanomenos, and distinguished it by the Greek letter Gamma, probably to perpetuate his improvement on their Diagrams.

The natural Hexachord had our C for its gravest sound, a fourth above Gamma or G, and the Moll, which had F for its gravest sound, was a fourth above C ; and when either of these Hexachords was found one or more octaves higher, the same name was continued, as being only a repetition of the former ; to which the seven Gregorian letters were added as before, except the Gamma, which was applied to the gravest sound G only. But as these letters were not sufficient to distinguish the mutations of the Hexachords, which were now to succeed in preference to the Tetrachords of the Greeks, something more was necessary to be done. For this purpose Guido added six syllables, taken from the first strophe of a hymn to St. John Baptist, beginning *Ut queant laxis*, &c. from which he selected the following syllables, and applied them to each Hexachord: Ut, Re, Mi, Fa, Sol, La. Of these, the Mi and Fa always distinguished the place of the semitone, and the other intervals, from Ut to



to Re, and from Re to Mi, Fa to Sol, and Sol to La, were always tones; but whether major, or minor, was not determined.

The Bb, or Moll Hexachord, is no new discovery, as it is formed on the same principles, and for the same purpose, in the Synemmenon Tetrachord of the Greeks..

The major mode, or scale, is principally described by this system of Guido; and the minor is rather implied than determined by any separate laws proper for a distinction of such great importance; and the whole seems calculated for the conduct of a voice in the first rudiments of singing, rather than a discovery of any one principle which can lead us to a knowledge of Harmony.

There is one discovery which must not be passed over, as it does Guido great honour, and has contributed much to the improvement of practical music.

It



It must be remembered, that the Gregorian letters, and the syllables which Guido added to them, were the only musical characters to determine the difference between acuteness and gravity. To prevent therefore the confusion which this must occasion, he is said to have placed several parallel lines at a small distance from each other, on which, and the spaces between, he marked little dots or points, by which the several degrees, from the most grave sound to the most acute, were soon discovered ; but as he found, that the extent of the natural voice could not be brought within the limits of four, five, or six lines, and that there were at least three species of voices, he applied the same number of lines and spaces to each, together with proper marginal characters or cliffs, to distinguish one from the other. Thus the gravest or bass voice was represented by the F Cliff, which confined it within the proper number of lines ; the mean, or C Cliff, was placed a fifth higher than the former, and its limits were equal to that species of voice ; and, lastly, the most acute or treble voice, which placed the notes a fifth higher than the mean, or C Cliff, was called the G Cliff.

By



By these improvements not only the acute, mean, and grave sounds were easily discovered, but also the species of voice to which they belonged.

Another part of his doctrine related to the mutations of the Hexachords, which depended principally on the position of the semitone. This interval was a material part in the Grecian Tetrachords, and by it they were conjoined to each other.

These are the chief parts of Guido's system; and, as they are to be met with in almost every writer from his time, and are so well known to professors, it is needless to be more particular.

He wrote a book, which he called *Micrologus*, to explain his doctrine; but, as I have never seen it, I can only observe, that if there had been any thing very material with respect to Harmony, the many writers in his favour would not have omitted to have taken proper notice of it; especially, as they have done him the honour of styling him the Restorer of Music, and the Founder of the modern harmonic system.

D

But



But how three simple Hexachords, or their Octaves, unsupported by the fundamental basses, (for as the writers since Guido take no notice of them, we must conclude they made no part of the theory, notwithstanding they are applied by the moderns;) how, I say, these Hexachords, whose intervals were undetermined by the ratio, could form a sufficient number of scales necessary for the mutations, together with a series of flat and sharp Diesis, proper to determine the place and magnitude of every interval necessary to perfect the most remote transposed scales, either major or minor, seems no less surprizing, than that of limiting the theories of the Greeks to a simple melody only of two Tones major and a Limma to each Tetrachord.

The truth is, we are convinced by demonstration, that Harmony has been discovered for some centuries past, but by what means, or by whom, is not so easily determined.

Vicenzo Galileo, who wrote A.D. 1581, says, page 80, that the discovery of composing or uniting several melodies was not more than 150 years prior to  
that



that time, which does no very great honour in respect to harmony to the sufficiency of Guido's system, which was in general use for so many years before.

Now, if Guido's theory was not capable of demonstrating the true principles of Harmony, to what shall we ascribe them that will not give the Grecians as good, if not a better, claim to the discovery, especially if we consider their great ambition of carrying every art and science to the highest perfection ; amongst which, Music was highly favoured, and men the most eminent for their learning made it an object of their study, and a part of the most finished education of their disciples.

I have now gone through the principal parts of Guido's system of Hexachords, which would have made no part of this work, had it not been with a view to shew how far his claim to the invention and improvement of Harmony can be admitted ; or whether, by an impartial and candid examination of the doctrine of the Grecians, they may not be found to have a better claim. This will best appear by their theories, which, as it is not possible to explain

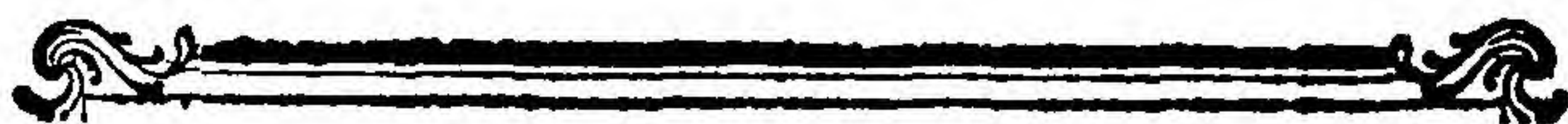


without the assistance and application of harmonic principles, (the only criterion by which we can judge) we must have recourse to harmonic principles, and thence may conclude in favour of the Greeks, by having the only proof on our side of which the nature of the subject is capable.



THE





THE  
THEORY  
OF  
HARMONICS.

PART THE FIRST.

THE following treatise being designed to illustrate the doctrine of the Grecians relative to the Harmonica, and to prove their claim to the true Principles of Harmony, it will be necessary to mention the Authors who have professedly written on the subject. They to whom I refer have been collected by the learned Marcus Meibomius, who has given an edition in Greek, with a Latin translation, and notes. [Amst. 1652, Elzev.]

Aristoxenus,



Aristoxenus, a disciple of Aristotle, the most ancient of these, has written three books, which he calls *The Elements of Harmonics*.

Euclid, the author of the *Elements of Geometry*, has written an *Introduction to Harmonics*.

Nicomachus, a *Compendium of Harmonics*; wherein he promised to explain himself more fully in his *Commentaries*. If he performed his promise, the loss of that work, among the several losses of the works of antiquity, must be regretted.

Alypius has written an *Introduction to Music*, and gives the various signs or characters by which the different degrees of tune were marked, peculiar to the several modes.

Gaudentius, the Philosopher, a favourer of the Pythagorean doctrine, has written an *Introduction to Harmonics*; in which are contained the two Diagrams that make the foundation of the second Part of this work, in which the Ratio will be applied to the Intervals.

Bacchius senior has written a short *Introduction on the Art of Music*, by question and answer.

Aristides Quintilianus treats both of the *Harmonica* and *Rythmica*.

These seven authors may be divided into two sects; the one following Pythagoras, in determining the difference of sounds,  
in



in musical intervals, by the Ratio ; by which is discovered the exact magnitude of each interval, consonant or dissonant ; the other, in opposition to the Ratio, making the Ear the supreme judge, as being more immediately concerned in the perfection of all musical intervals, and their succession.

As the physical properties of sounds, and the ratio of intervals, make no part of the theory of Aristoxenus, it is no wonder that the majority of succeeding writers embraced his doctrine in preference to that of Pythagoras ; and it is to this cause we must ascribe many difficulties we meet with in their works.

It required the most judicious and masterly contrivance to unite the different parts into which Aristoxenus's theory was unavoidably divided, and which were mutually to depend on each other, so as to establish a Theory independent of the Ratio, and at the same time capable, from the first definition of a sound, to comprehend and describe all the vast variety of intervals, which the different Systems, Genders, Tones, and Mutations demanded ; for, a deficiency in either of these essential parts would not only have destroyed the whole, but at the same time have fixed the Pythagorean doctrine on the most solid foundation.

The success which attended this opposition to the Ratio will best be understood from the principal rules laid down by these authors, which I shall collect and apply, without any forced construction, to the harmonic parts they were intended to explain. I proceed, therefore, to the Seven Parts into which the Grecian Harmonica was divided.

Euclid



Euclid gives this definition of Harmony; [p. 1.] that “It is  
 “ a science which contemplates the theoretical and practical  
 “ nature of a modulated series.” He adds, “A modulated  
 “ series is compounded of sounds and intervals, keeping a  
 “ certain order, and its parts are seven; of Sounds, Intervals,  
 “ Genders, Systems, Tones, Mutations, and the Melopœia.”

For the better understanding this general definition, I shall explain each part separately, with such observations as the several articles may require.

### I. OF SOUND.

“Sound is a pitch of the voice in one tension; [Aristox. p. 15.] or, as Nicomachus says, p. 24, “An indivisible voice.”

The ancients considered the voice as capable of two different species of motion. Euclid, [p. 2.] calls one, “the Continued, proper for Diction;” the other, “the Diastematic, “ (divided by intervals) proper for Melody.”

### II. OF INTERVALS.

Bacchius senior [p. 2.] says, “An Interval is the difference of two sounds, dissimilar in acuteness and gravity.” Aristoxenus [p. 15.] describes it to be “the space between two sounds, “ not having the same tension.”

Aristoxenus and Euclid give, though in a different order, these five divisions of intervals:

The



“The first, by which they differ among themselves in magnitude; the second, by which the consonant differ from the dissonant; the third, by which the compounded differ from the uncompounded; the fourth, by gender; and the fifth, by which the rationals differ from the irrationals.” Aristox. p. 16.

“The difference by magnitude (according to Euclid) is when some of the intervals are major and others minor.” Thus, ditone has a greater magnitude than trihemitone, and tone than hemitone or diesis: but diatessaron is less than diapente or diapasen, and the like.

The difference by consonance and dissonance is, that “the consonant intervals are formed by an agreeable mixture of the acute and the grave, but the dissonant refusing to mix, hurt the ear with some harshness.” Euclid, p. 8.

This mixture of sounds, signifying as consonance or dissonance, is a proof at least of two melodies or voices being united in the same song.

The difference of the compounded from the uncompounded is, that the compounded are such intervals as may be divided into less parts, such as the tone into two hemitones in the chromatic gender, and the hemitone into two dieses in the enharmonic; but in the diatonic gender, neither the tone nor hemitone can be divided, and are therefore uncompounded intervals.

E

There



There are, as Euclid observes, some intervals that partake of the nature of compounded and un-compounded ; as those from the hemitone to ditone : for hemitone in the enharmonic is compounded, because it may be composed of two dieses, but in the chromatic and diatonic it is un-compounded ; the tone in the chromatic is compounded, but in the diatonic is un-compounded. Besides these, there are other intervals which partake of these two opposite characters, by being applied to different genders.

“ The difference by gender is, when some of the intervals are diatonic, some chromatic, and others enharmonic.”

“ The difference of the rational and irrational is, that the rational are those whose magnitude can be given, as tone, hemitone, ditone, tritone, and the like ; but the irrational are those that vary these magnitudes, more or less, by some irrational magnitude.” [Euclid, p. 9.]

The preceding definitions of intervals are rather too general to give that satisfaction which so interesting a subject requires ; yet as they introduce the genders, in which the number and magnitude of all intervals less than the fifth or diapente, will be particularly described, they have their use, and prepare us to reconcile those judicious remarks and distinctions we shall meet with in the genders.

From these five divisions of intervals I shall proceed to the genders, to shew by what rule these intervals are to be discovered, and their different magnitudes determined ; for although  
the



the ear had before been declared the supreme judge, yet in some instances it was not sufficient; as Aristoxenus owns, in speaking of the enharmonic dieses, [p. 19.] where he says, “the ear scarce accustoms itself to this interval, even with much labour.”

### III. Of the GENDERS.

The genders will be particularly described by the number of parts into which Aristoxenus and Euclid have divided the tetrachord; by which division the magnitude of each interval is fixed and determined, according to the gender in which it is placed. And thus, not only all the imperfect concords, but even the smallest dieses, are discovered to have their particular magnitude within the limits of the tetrachord; together with those intervals which are proper for combination, and those which can be used only in succession.

The Tetrachord, not only from its great antiquity, but from the simplicity of its parts, is the principal foundation of the genders. Its composition consists of hemitone, tone and tone ascending, and the contrary descending. [Euclid, p. 3.]

By this definition we discover, that the hemitone is substituted in the place of the limma, which necessarily alters the two tones major into minor and major, which is truly diatonic.

That this may be better understood, it must be observed, that the composition of the tetrachord is limited to four sounds,



or three intervals, the extremes of which make the interval of a fourth or diatessaron, and are always in the diatonic gender fixed and immovable; but the two intermediate sounds are to be moved and altered, according to the gender that is to be formed. As these alterations will consist in making the middle sounds graver in one diagram, and acuter in the other, the modern characters of sharp and flat must be applied with the same effect, and in the same manner as the ancient dieses. These will be extracted in proper order from the several scales or diatessarons, which form the modes, and will be placed next each other as a scale, to distinguish and represent the spiss intervals, which though not proper for melody in that form, yet are absolutely necessary to divide the tetrachord into a certain number of parts, to determine the absolute magnitude of different intervals proper for each gender, and to form the different scales.

Euclid says, [p. 9.] that “Every melody is either diatonic, “ chromatic, enharmonic, common, or compounded of them.” And he adds, [p. 10.] “The mixt is that in which are perceived the characters of two or three genders; of the diatonic “ and chromatic, or diatonic and enharmonic, or chromatic “ and enharmonic; and even of the diatonic, and chromatic “ and enharmonic; and that the difference of genders is made “ by moveable sounds.” He multiplies these three genders into six, and adds, that “Colour is a special division of the “ gender, and that the rational and known colours are six; viz. “ one of the enharmonic, three of the chromatic, and two of “ the diatonic.” And after enumerating the particular composition of each, he tells us, [p. 11.] “that these colours are “ all shewn in numbers. The tone is supposed to be divided  
“ into



“ into twelve very small parts, each of which is called the  
 “ twelfth part of a tone ; and by the same method as the tone,  
 “ the other intervals are divided ; viz. a hemitone into six of  
 “ those twelve parts, the quadrant diesis into three parts, the  
 “ triental into four, and the whole diatessaron into thirty.  
 “ Therefore the enharmonic will be sung by the magnitude of  
 “ three of those twelve parts, and three, and twenty-four.  
 “  $(3 + 3 + 24 = 30.)$ ”

“ The chromatic moll, by 4 and 4 and 22,” = 30.

“ The sesquialter chromatic, by  $4\frac{1}{2}$  and  $4\frac{1}{2}$  and 21,” = 30.

“ The toniæum chromatic, by 6 and 6 and 18,” = 30.

“ The diatonic moll, by 6 and 9 and 15,” = 30.

“ The diatonic syntone, by 6 and 12 and 12,” = 30.

In order to explain this part of Euclid's doctrine, I must refer to Plate IV.

In this Plate the several intervals in each gender are regulated, and their magnitude determined, by the number of spifs or small intervals, contained in each, of which the following is an explanation.

Plate IV. K is composed of two tetrachords conjoined, to which the note of disjunction, marked by a  $\div$ , is added to compleat the diatonic octave.

L contains the spifs or small intervals, each being distant from the next, as directed by Euclid, three parts of twelve ; which divides the tetrachord into thirty such parts.

N con-



N contains the enharmonic gender; in which, after the two dieses are taken out of the tetrachord, of the magnitude of 3 and 3, the uncompounded ditone (consisting of twenty-four such parts, which are equal to the major third, an imperfect concord) is added, and completes the tetrachord in its proportion of thirty parts.

P is the sesquialter chromatic; but  $4\frac{1}{2}$  and  $4\frac{1}{2}$  are intervals not to be found in a continued series of 3; the interval, however, of seven dieses, or twenty-one parts, completes the tetrachord, and is an incomposite interval equal to a deficient fourth; which is an acquisition of great consequence, where all species of musical intervals are to be discovered and limited to their several magnitudes.

R, the toniaum chromatic, is composed of a semitone major and a semitone minor, to which an uncompounded interval, equal to a minor third, an imperfect concord, is added, which completes the number of parts equal to the tetrachord.

S is the diatonic moll; in which the two first intervals form the system of the deficient third, and the last uncompounded interval is the superfluous second. This gender, directed by a skilful composer, is very pathetic and sentimental.

T is the diatonic syntone. This gender is the scale of nature, consisting of semitone, tone, and tone. The perfection of this gender is such, that all, who sing, even without any knowledge of music, will sing their different melodies in this gender.



gender. It is not only the foundation of all the others, but by a judicious mixture with them forms all the colouring, or light and dark shades of music ; and without this, viz. the diatonic syntone, the other genders would rather disgust than please.

W, the chromatic moll, is involved in the same difficulties with the sesquialter chromatic ; for as neither 4, nor  $4\frac{1}{2}$ , are aliquot parts of 30, so no interval of these magnitudes can be found to have a place among the spifs, which are in a continued series of 3 equal to 30 ; and as no additional note can be placed among the spifs without exceeding the number limited by Euclid, and increasing the number of modes, which would confound and disorder the whole of their doctrine, so no alteration whatever can be made in the present number and order of the spifs ; and as the ratio was not admitted in Euclid's Introduction to Harmony, the difference of a comma cannot be applied on the present occasion, consistent with principles in opposition to the ratio.

For these reasons I omit any farther enquiry concerning the intervals at P and W ; particularly as those already discovered supply us with all the variety which every species of musical composition can want, either in succession or combination. For succession, we have the enharmonic dieses, the semitone minor, and deficient third ; and for combination, we have three species of 2ds, three species of 3ds, three of 4ths, three of 5ths, three of 6ths, and three of 7ths, with their respective octaves. These intervals are expressed at X and Z, Plates IV and V, and we discover in each (the one being an inversion of the other) three intervals



intervals proper only for succession, and seventeen, including the octave, for combination.

Thus the great end of the genders is in all respects answered, which was to fix and determine the magnitude of all species of musical intervals ; which could not be found in diagrams, composed only of tones major and limmas. But exclusive of these very great advantages, the genders contribute in a wonderful manner to the pleasure of those at all acquainted with them, by distinguishing the phrase, and giving a certain colouring to heighten and support the sentiment, and adding to the powers of mutation, in a more extensive manner than the diatonic alone can ever arrive at.

I cannot leave this subject without observing, that the great objection made by all writers to the Grecian theories has been, that they had only the perfect consonances, but that the imperfect made no part of their doctrine, and consequently it was impossible for them to have harmony or music in two or more parts, moving agreeably to the laws of harmonic combination.

This objection was founded on the construction of their diagrams, which were composed of tones major and limmas, intended by the Grecians to be applied in a very different way than in later times has been imagined ; and while that application remained undiscovered, the objection was not only true with respect to harmony, but was also true with respect to melody ; for such an untunable scale cannot be applied to either with any effect.

This



This great and universal objection will be removed when I consider those diagrams, and is at present in part removed by a right explanation of the genders ; an essential part of the Grecian harmonica, which can admit of no other construction, and has not only determined the magnitude of all the perfect and imperfect consonances, but of all the dissonances that can be discovered, or applied to musical composition.

#### IV. OF SYSTEMS.

It has already been remarked that any two sounds differing in acuteness and gravity form an interval ; that intervals are varied according to the gender ; and that genders are limited by the tetrachord to three intervals or four sounds ; by which limitation the magnitude of each degree is determined.

Thus in the diatonic the natural degrees of voice proceed from grave to acute by hemitone, tone, and tone, or the contrary ; and the whole composition of intervals forms the system of tetrachord.

Aristoxenus [p. 15.] and Euclid [p. 1.] agree “that a system is composed of more than one interval.” As this seems to be a definition of the smallest system, it follows that the two species intervals of the enharmonic and the chromatic must be systems ; and that the two species of thirds in the diatonic and chromatic may be made systems, by inserting a note between the extreme sounds of each third, as CDE, or DEF.

F

There



There are many species of systems described by the ancients, which, though they make a very considerable part, yet alone are not sufficient to supply the vast varieties of modulation. In aid therefore of this streight line of system, they have placed the uncompounded, or diastematic, intervals. By this method we discover two motions of voice; the first passing through the several degrees which form the system, and the other passing from one extreme of a system to the other, leaving out the intermediate degrees. Hence the diatessaron is of the same magnitude when a system, as it is when a diastem; and the only difference is that one is compounded, and the other uncompounded: therefore every interval in the diatonic, as diapente, diapasen, and the like, which admits of a composition, may be a system with, or a diastem without, such composition.

Systems then being a modulated series from grave to acute, or the contrary, are distinguished by Euclid [p. 12.] in the following order:

“ There are seven differences of systems; four of them are  
 “ in the intervals; namely, the difference of magnitude, of  
 “ gender, of consonance and dissonance, of rational and irrational.  
 “ There are three peculiar differences of the systems;  
 “ that of ordinate and preposterous, of conjoined and disjoined,  
 “ of immutable and mutable.”

The difference by magnitude and gender has already been treated of. The next difference, of consonance and dissonance, is distinguished by a comparison of the first sound with the last,  
 which



which terminates the whole, and may be sometimes consonant, and sometimes dissonant.

There are six consonant systems described by Euclid, [p. 12 and 13]. The first three are simple, as diatessaron, diapente, and diapasos; the next three are compounded of diapasos and diatessaron, diapasos and diapente, and bis-diapasos, which is the greatest and most perfect system, containing all the less. He also mentions a seventh and eighth system, viz. compounded of bis-diapasos and diatessaron, and bis-diapasos and diapente. Which shews that the limits of their scale were not so much confined as has generally been imagined; for it wants but one note only (the hypoproslambanomenos) to be of the same extent with that of Guido.

But Aristoxenus goes much further; for he says [p. 21.] that "A triple diapasos symphonizes, and a quadruple, and " even a greater magnitude."

Each of the first three systems is divided into different species or figures, by which, though their magnitude is the same, the parts of their composition are varied. This variation of the species is occasioned by the different position of the hemitone.

In the diatessaron it has three places, as  $\overbrace{BCDE}$ ,  $\overbrace{CDEF}$ ,  $\overbrace{DEFG}$ ; in the diapente it has four, as  $\overbrace{EFGAB}$ ,  $\overbrace{FGABC}$ ,  $\overbrace{GABCD}$ ,  $\overbrace{ABCDE}$ .



The several species or figures of the diapason are not only distinguished by the position of the two hemitones, but by the disjunction of the tetrachords.

Euclid [p. 14.] has been careful in distinguishing these different species. Thus from barypycni to the next barypycni above, or from B to E, is the first species of the diatessaron, [Plate I.] whose hemitone is in the grave. The second species of diatessaron is the mesopycni, or from C to F, whose hemitone is in the acute. The third species is the oxypycni, from D to G, whose hemitone is in the middle, consisting of tone, hemitone, and tone.

In this manner the three species of systems are explained; but as the extremes are all of the same magnitude, we can have but one diastem in the ratio of  $\frac{4}{3}$  to each, or from one extreme to the other. After these follow the four species of diapente, thus described by Euclid, p. 14, 15.

“ The first, that which is contained in the barypycni,  
 “ whose tone is first in the acute, and is from hypate meson  
 “ to paramese; the second, that contained in the mesopycni,  
 “ whose tone is second from the acute, and is from parypate  
 “ meson to trite diezeugmenon; the third, that contained in the  
 “ oxypycni, whose tone is a third from the acute, and is from  
 “ lichanos meson to paranete diezeugmenon; the fourth, that  
 “ contained in the barypycni, whose tone is a fourth from the  
 “ acute, and is from mese to nete diezeugmenon, or from  
 “ proslambanomenos to hypate meson. Also in the diatonic  
 “ gender, that is the first species (of diapente) whose hemitone  
 “ is



“ is first from the grave ; that the second, whose hemitone is  
 “ first from the acute ; that the third, whose hemitone is  
 “ second from the acute ; that the fourth, whose hemitone is  
 “ third from the acute.”

To these the seven species of diapason (mentioned by Euclid, p. 15 and 16 compared) succeed. They are called Tones or modes, very probably from the tone of disjunction, whose different position makes one of the principal distinctions of the several species ; and as they are composed of the two foregoing systems of diatessaron and diapente, the two semitones must have their place in each.

[Plate I.] “ The first is that contained in barypycni, from  
 “ hypate hypaton to paramese, (or from B to B) whose tone of  
 “ disjunction is the first from the acute, and whose hemitones  
 “ are first from the grave and fourth from the acute.”

“ The second species is contained in mesopycni, (or from  
 “ C to C) whose tone (of disjunction) is the second from the  
 “ acute, and whose hemitones are the third from the grave,  
 “ and the first from the acute.”

“ The third species is contained in oxypycni, (or from D  
 “ to D) whose tone (of disjunction) is the third from the acute,  
 “ and whose hemitones are the second from the grave, and the  
 “ second from the acute.”

“ The fourth species is contained in barypycni, (or from E  
 “ to E) whose tone (of disjunction) is the fourth from the  
 “ acute.”



“ acute, and whose hemitones are the first from the grave, and  
 “ the third from the acute.”

“ The fifth species is contained in mesopycni, (or from F to  
 “ F) whose tone (of disjunction) is the fifth from the acute,  
 “ and whose hemitones are the fourth from the grave, and the  
 “ first from the acute.”

“ The sixth is contained in oxypycni, (or from G to G)  
 “ whose tone (of disjunction) is the sixth from the acute, and  
 “ whose hemitones are the third from the grave, and the second  
 “ from the acute.”

“ The seventh is contained in barypycni and apycni, (or  
 “ from A to A) whose tone (of disjunction) is first from the  
 “ grave, and whose hemitones are the second from the grave,  
 “ and the third from the acute.”

These seven species of diapason are without doubt of the greatest importance. Euclid is not satisfied with describing them by the tone of disjunction, but goes over them a second time [p. 16.] to fix the positions of the two hemitones peculiar to each species. See Plate II. where the hemitones are described by the black notes, and the tone of disjunction is marked thus  $\dagger$ .

One tetrachord may succeed another by conjunction or disjunction.

The



The conjunction is made by a sound which is common to two tetrachords, and is the most acute of one and the most grave of the other.

The magnitude of this conjoined system is a heptachord or seventh, as from B to A, and consequently is a dissonant system from one extreme to the other. [Plate I.]

The disjunction is made by a tone, which separates the most acute of one tetrachord from the most grave of the other, and its magnitude is diapason, as from E to E, Plate I.

“ There are (as Euclid observes, p. 17.) two perfect systems ;  
“ the one less, the other greater.”

“ The less is by conjunction, from proslambanomenos to  
“ nete synemmenon, containing three conjoined tetrachords,  
“ hypaton, meson, and synemmenon, besides a tone from proslambanomenos to hypate hypaton, (or A to B in the grave)  
“ and is finished by the consonance diapason and diatessaron :”  
From A to the second D, or from the first to the second apycni, which he has made the limits of this less system. [Plate I.]

“ The greater system is by disjunction, from proslambanomenos to nete hyperbolæon ; and it contains four tetrachords,  
“ disjoined by pairs conjoined together ; namely, hypaton,  
“ meson, diezeugmenon, and hyperbolæon ; and two tones  
“ besides, one from proslambanomenos to hypate meson, and  
“ the other from mese to paramese, and is included in the consonance of bis-diapason, (from A to the second A) or from  
“ the



“ the first to the third apycni, which are the limits of the “ great system.” [Euclid, p. 18.] The great extent of this system discovers, that the ancients were not confined to the limits of diapason, as has been imagined.

It appears by the doctrine of the ancients, that the voice had two principal motions ; one, the diastematic or harmonic, when the sounds succeed each other by an uncompounded interval, as a 4<sup>th</sup>, 5<sup>th</sup>, or 8<sup>ve</sup>, and the like, proper for the fundamental bass ; the other motion of voice was by degrees or systems, which being more natural and better adapted to the singing of the voice, from their great variety, required a more critical description.

The diastematic motions are the only perfect intervals that can be discovered in a scale or diagram of tones major and limmas ; and as it is impossible to form a tunable system out of such a scale, it follows, that these uncompounded intervals are the true and original fundamental basses, and cannot be applied to any other part of a theory. As to the second motion of voice, in which the degrees or systems are formed, they are the products of those fundamental basses, by whose harmonic power the tetrachord conjoined, or disjoined, and all other systems are formed and regulated. See Plate III. and VII. in which mese, or the tone of disjunction, is marked thus +.

I must observe that all these systems are formed by the tetrachords, and not from the diagram of tones major and limmas ; for Euclid, p. 8, and Aristoxenus, p. 20, say, “ that all intervals less than the fourth, or diatessaron, are discords.”

This



This is very true if applied to the diagram of tones major and limmas, in which no consonant system can be found ; but with respect to these tetrachords, which are all truly diatonic, it cannot be applied to them. In order to remove this seeming contradiction of intervals, or even systems, less than the diatessaron being discords, and to give all possible satisfaction on this subject, I must observe, as before, that the Grecian theory was divided into two parts. The first and principal was the perfect system, or diagram of tones major and limmas ; from which system all harmony with its roots or fundamental basses arises. The second part related to melody, which is connected with, and inseparable from, the fundamental bass. Thus the scale of tones major and limmas appears to be expressly formed to regulate the fundamental basses, and cannot be applied as consonant systems, like the tetrachords, nor indeed to any other part of their theory.

The harmonic parts, from whence melody has its original, are found in the tetrachords, systems and genders, &c. and are the products of the fundamental basses. For a full explanation of which see pages 50, 51, &c.

In the scale of tones major and limmas, the only consonant sounds to be found are the diapason, diapente, and diatessaron. These three consonances have a very distinguished character by being applied to each tetrachord as fundamental basses ; for they not only discover their true and harmonic powers in forming the diatonic octave or scale, (all of which may be extracted from among the harmonics, and reduced to a system of degrees) but the name of the scale or key-note, when formed, will

G

have



have the same name as the principal consonance, or diapason. This concord is not only the original of the other two, by including them within its limits, but is superiour to either by being the final note of cadence, and all other perfect terminations. The diapente is more limited in its powers as a fundamental bass in this respect, that it forms the imperfect repose, and is necessary to lead to the final by forming the cadence.

The Diatessaron is one of the primary fundamental basses, and contributes with the other two to form the scale, or diatonic octave, but is incapable of forming any termination; it has however one character peculiar to itself, that of being the first and original discord, as its relation to diapente is tone major; yet, with this application, it becomes an harmonic sound, as all original discords are, and not a fundamental bass.

Having discovered the great importance of the scale of tones major and limmas, in being the original of the three fundamental basses applied to each tetrachord, I shall now distinguish them in another point of view, in order to facilitate their use. I shall place them as antecedent, mean, and consequent, or as first, second, and third terms. In this form, the diatessaron, or fourth, will be the first; the diapason and most perfect consonance will be the key note, or second term; and the diapente, or fifth, will be the third term, attended with their respective harmonics of an octave, fifth, and third. It must be remembered, that all systems have their foundation in these three terms, and that as they will be frequently applied in explaining the tetrachords, it is unnecessary to add more here. I will therefore proceed to explain one of the most curious and  
interesting



interesting particulars discovered in the whole theory of harmonics, namely, an inverted system, which will be found of the greatest consequence; for, by the inversion of the first diagram a second is discovered.

Euclid [p. 15 and 16.] places his seven modes in the following order, and Bacchius Senior [p. 12.] places his seven in a contrary or an inverted order, and the same names are applied to each.

Euclid.	{	B	—	—	Mixolydian	—	—	—	F	} Bacchius Senior.
		C	—	—	Lydian	—	—	—	E	
		D	—	—	Phrygian	—	—	—	D	
		E	—	—	Dorian	—	—	—	C	
		F	—	—	Hypolydian	—	—	—	B	
		G	—	—	Hypophrygian	—	—	—	A	
		A	—	—	Hypodorian	—	—	—	G	

Though this inversion of the diagram may appear at first sight to be of little consequence, yet it has engaged the attention of many writers, and occasioned as many different opinions, without producing any thing decisive; for whether the diagrams are to be read backwards descending, or the contrary, or whether the grave sounds are called acute in one age, and the contrary in another, is of very little consequence, where an art is to be improved, or the secrets of science to be discovered. But I shall pass by these and other favourite opinions, which have only served to perplex the subject, without leading us to any useful discovery.



To procure, however, all the advantages this doctrine can give us, a musical construction, supported by other parts of the Grecian harmonica, will shew its real importance. In order to demonstrate this I shall observe, that the seven species of diapason [Plates III. and VII.] include all the tetrachords in their different positions, whether conjoined or disjoined, and also the hemitones, tones, and all other necessary distinctions proper for harmony or melody; which will be explained in the next or fifth part, in which the modes or tones will be particularly considered. At present, therefore, I will offer a few observations, previous to my entering on so interesting and difficult a subject.

I observe first, that every system of diapason is called a tone or mode by Euclid and Bacchius, and that the first letter of that diapason is placed near the name, which is given to the mode.

Secondly, That Euclid places the mixolydian at hypate hypaton, (or B) in the grave, and ascends by degrees till he arrives at mese, (or A) in the acute; whereas Bacchius places his first, or mixolydian mode in the acute, (or at F) from whence he descends by degrees to the grave (or G); and each author distinguishes every succeeding mode by the very same names, but in an inverted order; as in the preceeding example, at page 43.

Thirdly, That in comparing Euclid's diapasons with those of Bacchius, the intervals are all the same; that is, the tones hemitones, and tone of disjunction, are in the same position in each, except being in a contrary direction. See Plate VI.

Fourthly,



Fourthly, That Euclid, [p. 19.] (according to Aristoxenus) affirms, that "the number of tones or modes are thirteen," but that number cannot be supplied, unless they are distinguished as two separate scales; the first ascending from grave to acute, the second descending from acute to grave.

Fifthly, That, agreeably to the position of the seven species of diapason, at Plates II. and III. we discover all the sharps in a numerical order from one to six.

Sixthly, That the same seven species of diapason being inverted, we have all the flats from one to six. See Plates VI. and VII.

Seventhly, That these sharp and flat dieses form the spiss intervals, the minor and major scales, the genders, the mutations, and divide the tetrachord into thirty parts, as directed by Euclid.

Lastly, That the fundamental basses, or three terms before mentioned, applied to each tetrachord, form a most sweet and perfect harmony.

These are the great outlines of a most extraordinary theory, which will be explained and applied in the next or fifth part of the harmonica; in which the tones and number of modes will be more particularly considered.



## V. Of the TONE or MODE.

This part of the ancient doctrine has been found the most obscure and mysterious. The many learned writers and commentators, who have attempted to explain it, have disagreed in many essential parts, whilst others, from the variety of opinions, despaired of success, and gave up the enquiry. The truth is, the wonderful and extraordinary accounts we meet with in ancient and modern writers, of the surprizing effects produced by the mutations of mode and gender, and other parts of the Grecian harmonica, have raised expectations in many, not to be satisfied with any thing less extraordinary than the effects said to be produced. Whereas, had they first considered what were the essential parts of practical music, and adapted them to the ancient theories, without expecting any thing almost supernatural, they would have found the agreement between the ancient and modern methods of treating the same subject so perfectly corresponding in the most essential parts, that nothing could be discovered in one, that would not be wanted and absolutely necessary in the other.

If a practical musician were to be limited to the four parts already explained, he would find a great deal wanting to enable him to exercise the full powers of his art. He would be greatly distressed in his operations, whilst confined to those sounds only which have been discovered. It is true, the genders have explained the division of the hemitone and tone, and other musical intervals, but have given no rule to apply those divisions to any other part than the two moveable or middle sounds of the tetrachord.



tetrachord. Something more therefore is necessary for a general application of those divisions, by which modulation may be carried to its greatest perfection, without any alteration or injury to the tetrachord. And if that which is so indispensably necessary for modulation can no where else be found but in the doctrine of the tones and modes, and if there is no other part of practical music to which they can be applied, let us not reject so interesting a part of ancient theory, without at least trying how far it will carry us.

Euclid [p. 19 and 20.] gives four different definitions of the word *Tone*.

He says, "It is taken for sound;" as a sweet or harsh tone or sound.

Secondly, "It is taken for interval; as from mese to paramese (or from A to B) is the interval of a tone."

Thirdly, "It is applied to the place of the voice, when we mention the dorian, lydian, or any other tone."

Fourthly, "It is taken for intension or remission, when a person uses an acute, or grave, tone of voice, or between both."

The third definition, or "the place of the voice," is the subject of the present enquiry.

All



All authors agree, that the seven species of diapason are the foundation of the doctrine of the tones or modes, and that the tones to which they refer are not to be understood in the limited sense of sound, interval, or intension, but as a certain composition or constitution of harmonious sounds, proper for the place of the voice in the most extensive sense; provided the two hemitones and the tone of disjunction are placed according to the method described by Euclid in treating of the system of diapason.

That this method may be more clearly understood, I have (at Plate II. fig. 1.) formed the seven tones or modes, agreeably to the different species of the diapason, in which the positions of the two hemitones are distinguished by being black notes, and the tone of disjunction by the cross, marked +, between the two disjoined sounds. The tetrachords are also distinguished by the curve line, which encloses the whole, with the extreme sounds, and the provincial name is annexed to each tone, as mentioned by Euclid.

In examining these constitutions, which are the foundation of the modes, we find the position of the two hemitones and the tone of disjunction the same in each with respect to acuteness or gravity; that is, from B to C, and from E to F, are, in all of them, the places of the hemitones, as from A to B is also that of the disjunction. Now, as the different position of these intervals should make the essential parts of modulation, which notwithstanding are the same in each, to which of the remaining parts of these compositions must we turn our attention to discover something characteristic, and expressive of the  
different



different relations, in which one tone or mode stands to another? Not in the several limitations of the diapason; for that will be a positive contradiction to a former part of the doctrine, which gives a minor system, consisting of a diapason and diatessaron, and also a major system, composed of twice diapason. Now, if a voice has liberty of motion through all, or a great part of this major system, what end can it answer to confine it to half that compass? or, what new variety of modulation will it procure, which we had not in a much greater perfection before? But this is not the only objection; for the synemmenon tetrachord, which gives a hemitone from mese to trite synemmenon (or from A to B $\flat$ ) in a different position from the two former, is absolutely excluded from a place in each of these seven species of diapason.

However great these contradictions may seem, they ought not to be imputed to the ancients, unless it shall appear that these seven species of diapason can admit of no other construction.

But the contrary of this will be evident, if we attend to the definition which Euclid has given; [p. 2.] wherein he tells us, that “tone is a place of the voice, capable of system without latitude.”

This definition is very important; for it not only determines as many different places of the voice as there are systems of the diapason, but also that each system is without latitude. Thus it appears, that the diapason is no otherwise necessary with respect to the limits of the system, than its first formation,

H

by



by which the two hemitones and the tone of disjunction, have each their particular positions; for these are the only essential differences of the seven systems or places of the voice, after which the systems are without latitude; that is, the systems may extend beyond the limits of diapason, either to the grave or the acute, provided that the positions of these intervals are the same in the respective diapasons, above and below; for the order and composition of the first diapason being once fixed, determine all others belonging to the same tone, and are properly that which Euclid calls "the tone or place of the voice" capable of system without latitude;" because every tetrachord found in this composition refers to one particular sound or tone, which forms one scale only, and no other.

After this explanation it remains to shew, how these seven places of the voice may be discovered agreeably to these directions; and how such discovery can encrease the powers of modulation, by disposing of the two hemitones, and tone of disjunction, in seven different places with respect to acute and grave.

To make this easily understood, I have, at Plate II. fig. 2. placed the seven species of diapason in the upper stave, each species rising a degree in the order as before, at fig. 1. in the same plate, and immediately under these I have placed seven others, at H, I, K, L, M, N, and P, whose compositions are in all respects the same, except that of being formed from one common sound, instead of rising a degree, as those above.

This



This common sound is B, the first of the gravest tetrachord, and in that respect has a preference to any other. From this construction we have a variety of hemitones distributed throughout, not discovered before; and as these intervals are the same with those in the stave above, the sharp, which is a modern character, is applied not only to fix the positions of the tones and hemitones peculiar to each mode, but also to shew their relation to each other, in order to perfect their mutations; without which the whole theory would have been imperfect.

Thus after the natural system at H, we have an increasing series of sharps from one to six. The first sharp is F, the fourth species of Diapason at I; we have two sharps, F and C, in the seventh species at K; and at L we have three sharps, F, C, and G, which is the third species; and at M, the sixth species, we have four sharps, F, C, G, and D; also at N, the second species, we have five sharps, as F, C, G, D, and A; and at the fifth, or last species of diapason, at P, we have six sharps, viz. F, C, G, D, A, and E.

In comparing these seven species of diapason, ascending from one pitch or common sound, with each other, we have five tones, each of which is divided into a greater and less hemitone; also one hemitone, divided into a hemitone minor, and an enharmonic diesis; the first from E at H, to E # at P; and the second from E # at P, to F at H, in the first species. With respect to the other hemitone, from B to C, it could not, according to this method, be divided, because B is the pitch, or common sound; but this is not material, as we shall, in another place, find it divided in the enharmonic gender, Plates IV. and V.



These different positions of the two hemitones and the tone of disjunction give every diapason a new and distinct character; which, with the other advantages before enumerated, explain the ancient doctrine of the tones or modes in the most satisfactory manner; and, by their agreement with Euclid's definition, leave no doubt that "the tone, or place of the voice, capable of system without latitude," is found in each of these seven species of the diapason; because, the constitution of the diapason being once formed, may be extended to the acute or grave in the same order, without latitude.

By this construction the most important part of the harmonica, with respect to modulation, is reconciled with the doctrine of the ancient modes, in a manner the most natural and useful to a practical musician.

The Tone of Disjunction, so often mentioned by the Grecians, and found in every diapason, comes next to be considered, in order to discover its character and use in this theory.

If we examine the seven species of diapason, where this interval is marked with a cross, we discover very little that can make this distinction necessary, except that of fixing the limits and order of the tetrachords; but as this seems of too little consequence for such a distinction, if confined to a simple melody, we must look for its true character and importance in harmony.

To shew this in the clearest light, I have (at Plate III.) repeated the seven species of diapason, formed from one common  
found,



found, as before; and have placed under each note its principal or fundamental bass at R and S.

I observe at R, the first species of the diapason, we have two tetrachords conjoined, and the tone of disjunction succeeds. That each of these tetrachords has three basses; the first has E, A, and D; the second has A, D, and G; and finishes at D bass. After which we are prevented from advancing to the tone of disjunction, unless the bass, and its harmonics at D, move first to those of A; then the tone of disjunction and its bass succeed. It appears also, that every bass in this diagram must have a minor third for one part of its harmony; in consequence of which, the B, in the first tetrachord, will be of a different species from that in the second; for the first tetrachord begins with a B natural, and the second demands a B flat, to be a minor third to G, the third bass.

From this observation we discover that these tetrachords are formed so as to partake of two distinct scales; as it is impossible to have a natural B and a flat B in the same diapason. Thus by applying the fundamental basses and their harmonics we discover that the true character of the tetrachords has its origin in harmony, by which their formation and connection appear rational and agreeable to the best theory; which will be more fully explained when we consider the Mutations.

The tone of disjunction distinguishes the termination of the last tetrachord, in which B was necessarily flat, and restores us to the first scale, in which it was natural; but in order to prevent the disagreeable effect, from too quick a succession of a B  
flat



flat and a B natural, the intervention of an additional bass and its harmony is required, as marked at 8, (between the two fifths at R) previous to the disjunction ; this not only conducts us to the alteration of the scale, which is here intended, but prevents the consecution of two fifths, which in no case can be admitted. See Plate III. at R.

From these observations we discover the real importance of the tetrachords ; that they are not to be considered merely as simple systems, but as systems connected with, and dependent on, the laws of harmony.

I shall now compare the same species of the diapason at R with those at S, in the same plate.

In these two examples we find no alteration whatever in the systems ; for the two hemitones, the conjoint tetrachord, and the tone of disjunction, are the same in each ; and the only difference discovered is in the basses, which is undoubtedly a very material one with respect to modulation.

To understand this it must be observed, that the systems now before us at S are not tetrachords, but are a species of the diatessaron ; for the true position of the tetrachord, in a major mode, must be in all respects an inversion of the minor ; which Euclid has before told us is hemitone, tone, and tone ; agreeably to which, the hemitone must be in the acute in the major scale, as it is in the grave in the minor.

If



If we compare these systems at S with the tetrachords at R in the same diagram, or those at U with T, Plate VII. we shall find that two fundamental basses are sufficient for these systems, but that the tetrachords require three.

This distinction would have been of very little consequence, if the formation of the scale did not depend on the harmonious constitution of the tetrachord, which cannot be extracted from the harmonics of two fundamental basses, but is the product of three ; for they are placed in this order not by accident, but to shew that the same melody may have two distinct characters, by being formed from different fundamental basses ; and that without their application the minor and major scales must remain undetermined.

It has before been observed, that the basses at R required minor thirds ; but these at S, on the contrary, do demand major thirds. This variation of the thirds occasions one of the most remarkable revolutions to be found in harmonics, and is the original cause of the mutations from minor to major scales, or the contrary. As to the difference of tone major and minor, which may be found in these systems, the ratio making no part of the Aristoxenian doctrine, I shall reserve my remarks on that subject for the Second Part of this work.

With respect to the systems now before us, we find but two basses to each at S, instead of three, which are necessary at R ; likewise that the tone of disjunction at S requires an intermediate bass, D, to prevent the succession of two major thirds, equally to be avoided with the two fifths at R ; besides which,  
the



the major third applied to this, and to the other intermediate basses at S, marks the change of the scale occasioned by the disjunction.

I have now gone through my observations on these seven species of diapason, so far as they relate to the explanation of the tones or different places of the voice. Their connection with harmony, and their mutations are supported by the fundamental basses, and an increasing series of sharps; and the distinctions of tetrachords conjoined, or disjoined, are made manifest, and supported by the most unerring laws of harmony.

The advantages arising from an increasing series of sharps are, it must be owned, very great; yet a Theory would be very imperfect if it should stop here and leave out the flats, which are equally necessary for the completion of the whole.

The flats being of a contrary nature to the sharps, require the whole order of systems to be inverted; but nevertheless the place of the tetrachords, the hemitones and tone of disjunction, must be preserved in the same positions before described, except that of moving in a contrary direction: but as this will best be understood by the musical notes, I shall refer my reader to Plate VI. fig. 1. and 2. in which the two diagrams are formed on this principle of inversion.

The first, which has already been explained, moves in an ascending, but the second in a descending, direction; placing the hemitone in the acute, from whence the tetrachord begins in a major scale.

In



In comparing their intervals, and other parts, we find a mutual agreement throughout the whole.

The seven species of diapason, extracted from the second diagram, have their beginning at fig. 3. Plate VI. and are continued till all the seven are formed.

Each diapason in the upper stave descends a degree to the next; but those immediately under are fixed to one common pitch at F; but in all other respects the intervals are the same with those above, and move in the same direction. Hence we have an increasing series of flats from one to six, which fixes and determines the positions of the hemitones, tone of disjunction, and the tetrachords, extracted from this second diagram; and the whole agrees with the first diagram before explained, whose common pitch was B, from whence this inversion is taken. See fig. 4. Plate VI. which exhibits the seven species of diapason belonging to each of the diagrams, above and below; the flats succeed and increase in the same order below, as the sharps do above. Example.

Sharps.	F #,	C #,	G #,	D #	A #,	E #,
	1,	2,	3,	4,	5,	6,
Flats.	B b,	E b,	A b,	D b,	G b,	C b.

The reason why B # and F b are not placed here will be given towards the latter end of the Mutations.

From the great advantages procured by the construction of these two diagrams we have all the materials necessary to complete the seven parts, into which the Grecians divided the har-



monica: at the same time the powers of harmony operate so universally throughout the whole theory as to explain many parts which could never be understood upon any other principle. Among which are the conjunction and disjunction of the tetrachords, with the major and minor harmonics applied to each tetrachord, or system, of the second diagram, at Pl. VII, fig. T and U. The seven species of diapason, formed under each of these letters have their fundamental basses to which they severally belong, with the major thirds to the first at T, and the minor thirds to those at U. The tone of disjunction is distinguished in each by an additional bass as a temperament, not only to prevent the consecution of two eighths at T, and two minor thirds at U, but also that the fundamental bass should not move by degrees or partake in any respect of the nature of the system, which is set apart, and appropriated only to the melody. The additional bass at U generates a discord at the disjunction with the upper sound: but this shall be explained in the Second Part, when I treat of Discords.

From the principles already explained of the harmonica of the Grecians, it appears that the great variety of intervals, their relation, and the different orders of their succession, together with their particular application to musical composition, are the principal objects intended to be regulated by these several parts of their doctrine.

With these materials I proceed to the Mutations, which is the sixth, and most interesting, as well as the most difficult, part of the harmonica. The principles laid down by  
Euclid



Euclid will conduct us through all their charming varieties : at the same time the formation and number of modes will be discovered, which could not be explained so well before, as they depended much on the mutations. We shall then be convinced, by the most indisputable evidence, of the agreement between the several parts which compose and perfect this wonderful theory.

## VI. Of the MUTATIONS.

To explain how the Mutations are to be made, it will be necessary first to discover their object, and then the method by which they may be effected, so as to produce what Aristoxenus declares mutation to be, “ A kind of pathos, occurring in the “ order of modulation.” [p. 38.]

In explaining the tetrachord on harmonic principles, it has been found that the several sounds, selected from the harmonics of a tetrachord and placed as a system, form the diapason ; that they are a scale of sounds, peculiar to that tetrachord, and are not found in any other ; and that whatever flat or sharp dieses are used in the tetrachord, the same are absolutely necessary for the formation of the system of the diapason, or scale.

The ancients very judiciously placed the mutations in the change of the tetrachord ; not only because the several genders would be better understood, when applied to that small system, but because of its harmonic principles, (before explained) ; which they knew comprehended the whole scale or diapason, by



selecting its several harmonic parts, and forming them into one system.

By this harmonic construction of the tetrachord its great importance is discovered: and if compared with that description which we meet with in all authors who have written on this subject, we shall find every part unlike, except a simple system of four sounds, unconnected with harmony, and independent of every principle that can lead us to the knowledge of it. No wonder therefore that their diagrams have been exploded and their theories condemned.

If we examine the two tetrachords in the first species, and those which make a part of the other species of diapason, (at Plate III. fig. R and S) they, by having major or minor thirds to their respective basses, contribute to form different scales. By these different species of thirds we discover their genius and character to be very unlike. The first at R gives two scales with minor thirds, because each tetrachord forms a scale from among the harmonics peculiar to itself. Notwithstanding the intervals of the diapason are in appearance the same in each at R and S, yet the harmonics are in all respects very different by the application of the fundamental bass, which not only forms two scales of different species of thirds, but also fixes their distance from each other to be a minor third, with the same number of dièses, either sharp or flat.

With respect to these scales, every one has a principal, or governing note, to which all the other sounds refer, and stand in a particular relation.

This



This principal note regulates the intervals, position, and motion of all the other sounds in that scale; and, by a kind of attraction, draws them to itself, or its harmonics, to finish the termination on the final place of repose.

This principal note is distinguished by the moderns as the key-note, or the tone; and sometimes in a more enlarged sense, both by the ancients and moderns, the Mode. But in order to be better understood in this very difficult part of the harmonica, I shall apply the words key-note, or tone, to the principal note of a simple scale; and the word *Mode* to comprehend several scales, whose situations, or relations, to the first or governing scale are such as open a ready and easy communication for the mutations; which are made by substituting one scale in the place of another, or by moving from the principal to any of the relative scales, or the contrary: by which means the sentiment is continually varied, and the same thought appears in a new light by being removed into a different scale.

Thus these additional scales, subject to certain limitations with respect to the principal or governing scale, constitute a mode, in the most extensive sense of the word; for as all the sounds in a scale refer to, and depend on, the principal sound, so all these secondary scales in like manner refer to, and depend on, the first, as a principal or governing scale. And in this sense every principal sound of the different scales in each diagram may, by a like addition of secondary scales, be formed into a mode; and although this mode is described by the principal sound of the first scale, as D major, D minor, &c. yet all the secondary  
scales



scales are understood to be connected with it, in as ample a manner as if they had been severally named. In order therefore to discover this principal note, the following observation will serve as a rule :

The fundamental bass to the last note of each tetrachord is the principal note: and the moderns call the scale by its name.

Thus in the first diapason at R, (Plate III.) we finish the first tetrachord at A, the fundamental bass; and therefore the whole composition of that tetrachord forms the scale of A, as the next tetrachord does that of D scale.

The disjunction, or conjunction, makes no other difference than that of obliging us to continue the names of the notes in the same order till the whole tetrachord is completed; and under the last note we find the bass, which gives its name to that scale.

Thus at the second species of diapason at R, the note after the disjunction begins the tetrachord at A $\sharp$ , and though B is the last note of that diapason, yet it is not of the tetrachord; and therefore to find the principal note we must return to C $\sharp$  and D $\sharp$  in order to finish the tetrachord; and the bass G $\sharp$  is found under the last, or D $\sharp$ , and is the principal found which gives its name to that scale.

But



But that this may be better understood, I will examine and make some observations on the harmonic constitutions of the tetrachord in the minor and major scales, as formed by the fundamental basses in each diagram.

<p>W. { <math>\overbrace{B, C, D, E,}</math> 1st Tetrachord.  <math>\left\{ \begin{array}{l} G, A, A, A, \\ E, E, F, E, \end{array} \right.</math>          Roots E, A, D, A.</p>	<p>X. { <math>\overbrace{F, E, D, C,}</math> 2d Tetrachord.  <math>\left\{ \begin{array}{l} C, C, B, C, \\ A, G, G, G, \end{array} \right.</math>          Roots F, C, G, C.</p>
-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

In each of these tetrachords we find that in four combinations the fundamental bass, which gives its name to the key or scale, is used twice in each, viz. the second and last bass at A in the first, and the same at C in the second tetrachord ; besides which, a repetition of the same key-note is discovered among the harmonics in three combinations out of four, in order that the impression of the principal sound and its harmonics, with which the ear should be most familiar, may predominate, and be preserved without any improper mixture of other sounds.

From these two tetrachords B, C, D, E, and F, E, D, C, the minor and major scales are extracted : and, except the difference of the pitch, the intervals are the same in each, without any sharp or flat diesis, or any distinction to support the idea of minor and major scales, till we apply the fundamental basses as above ; which is the only rule or criterion by which we can judge of the difference : for the same melody or system may be formed in either scale, by applying different roots. See Plate III. at R and S, where the roots at R have minor thirds, and the same system at S has major thirds to each root. This, together



together with other instances, proves that the theories of the Grecians were founded in harmony; without which their minor and major scales must have remained undetermined. The two tetrachords at W and X are in their harmonic parts most unlike; for, exclusive of the observations already made, it appears that A, the principal or governing note of the first tetrachord at W, is used but once at X in four combinations, and is then a major third. In like manner it appears that C, the principal or governing note of the second tetrachord at X, has only one place at W in four combinations, and is then a minor third. These alterations preserve the character of each, and remove every possibility of mistaking one for the other; and at the same time discover the great order in which these operations are conducted, and the different effects they produce. If the Aristoxenians had not excluded the ratio from their theory, the difference of the major and minor tone, and other intervals, would have been explained by them. But this will be considered when we come to treat of the Ratio in Part II.

All tetrachords are formed on the same principles as those above, and their several positions, as at Plates III. and VII. discover the increase of the flat and sharp dieses, necessary to perfect every scale, even in the most transposed position. And hence we may conclude that the seven species of diapason, which are the foundation of the modes, could not be intended, as some imagine, to discover new scales of a different constitution, but only to change the position of those already existing, in order to improve and perfect the mutations. It is true that the enharmonic and chromatic dieses are described in their genders; and by particular divisions of the tetrachord their  
magnitude



magnitude is ascertained; but as this alone was not sufficient, where so extensive a distribution of intervals is wanted for the formation of so many scales, something more was necessary to increase those animating powers, essential to the mutations.

The different species of diapason, as before explained, answer this great end in every respect; for the formation of all scales out of the natural not only depends on the application of a flat or sharp diesis to alter some note a semitone higher or lower, but as we proceed with the diapasons, the number and position of the dieses, or flats and sharps, are fixed and determined from one to six in each diagram. By this method the major or minor scale is known the instant the number of dieses is discovered, and the absolute pitch of the key or tone naturally follows. But that this may be attended with as little difficulty as possible, I have formed all the scales from the two natural at A and C to the greatest number of sharp and flat dieses that can be wanted to complete the modes, even in their extreme position, by which all possible mutation is perfected. See Plates VIII. and IX.

In examining these scales we discover that both the sharp and flat dieses increase in a numerical order by a series of fifths from the natural scales A and C, extracted from the first species of diapason; and that as the sharps ascend by fifths from these natural scales, so the flats descend in a contrary direction by fifths; and as there are major and minor scales peculiar to each diesis, they are enclosed in a bracket in the margin, together with the names of the scales, the number of dieses necessary for their formation, and the species of diapason from whence they are extracted. The three terms, or fundamental basses are also

K

placed



placed at the end of the scales, as being the foundation from whence they are formed. The composition of the tones and semitones is the same in all the major scales. The minor scales compared with each other have the same composition ; but if compared with the major, there is a very essential difference in the position of the tones and semitones : the first is distinguished by being open notes, and the second by being black in all these scales.

There is also another difference with respect to the ascending or descending of the major or minor scales. In the first there is no alteration in moving either way ; but in the second there are two accidental major sounds ascending, which are not necessary in descending. The preserving of a diatonic succession occasions this difference in descending and ascending in the minor scales.

These scales being explained, I proceed to their use and application, by which that most captivating part, the mutations, will be better understood.

The words tone and mode have by some modern authors been used as synonymous terms ; but, as I have observed before, Euclid tells us, [p. 2.] that “ tone is a system without latitude ;” that is, that the same system may move to the acute or the grave, without any limitation, provided the tones and semitones succeed in the same order, as in the first diapason or original scale.

From



From this definition it appears that every scale may be a tone, and have its principal note or key as marked in the margins of Plates VIII. and IX. but with respect to a mode, it is formed of several of those scales, subject to certain laws and limitations, as has been observed before; without which it would be impossible to have either a consonant or dissonant mutation, or any other, agreeable to the directions of Euclid; who, after enumerating four sorts of mutations, says [p. 20.] “ the mutation by tone is when we make a change from dorian  
“ to phrygian; or, in general, from any one of the thirteen  
“ tones into any other; and these mutations are made beginning  
“ from hemitone to diapason.”

From this description of the mutations, proceeding from hemitone to diapason, it appears that when the diapason, or principal tone of the mode, is formed, the mutation is made by an addition or subtraction of one diesis from the original number. Hence we discover that the mutations depend on the increase or decrease of the dieses. If we examine the scales at Plates VIII. and IX. it appears that any number of dieses forms two scales; one with a major and the other with a minor third. In consequence of this we have a choice in fixing the principal tone of a mode in either of these scales; and though the number of relative scales will be the same in each, yet those of the same species of third with the principal tone of the mode must, as being primary scales, have a preference in the mutations to those of a different species, which are of a secondary nature, and are used in a transient way, to give a colouring, and vary the lights and shades of the mutations



As three terms in the major scale have the same species of thirds, they determine the names of the three primary scales of the mode, one of which is a fifth above, and another a fifth below the governing or principal tone. The other three fundamental basses, which have a different species of thirds, describe the names of the secondary scales of the same mode. We discover from hence that every mode is, or may be, composed of six scales; viz. three with major, and three with minor, thirds; and that the pitch of the governing or principal tone is the name of the mode, and includes its five auxiliary scales, from which the mutations are made.

The harmonious constitution of a mode being thus explained, it is proper to observe that from the number of scales we may fix the principal tone or pitch of the mode higher or lower, in order that every species of voice and instrument may be accommodated to move within the limits best adapted to its natural powers. But this makes no alteration in the composition of the mode, whose harmonic parts, as well as the auxiliary scales to the principal tone, are the same.

The inversion of the tetrachords in the second diagram, Plate VII. being explained in a former part, and several observations made on the conjoined and disjoined tetrachords, I proceed to examine the extensive powers of the fundamental basses taken from the great and unchangeable system, which is the foundation of the tetrachords, and all other systems.

The principal motion of the fundamental basses is the interval of a fifth, of which the fourth is the inversion; it being the  
same



same whether we move from C to F five descending, or four ascending, as shall be most convenient in writing to bring them to their nearest degrees.

It has been found that every system of the tetrachord is the product of three terms or fundamental basses; and on examining the motion of the basses it appears that two of the harmonics move with the bass, and the third harmonic is common to two basses, and is a ligature; but of the two sounds that move, one is always a tone, and the other a semitone. See p. 63.

This is the harmonious constitution of the tetrachord; and the diatonic scale, or system of the diapason, peculiar to that tetrachord, is formed by extracting from the harmonics such sounds as are proper for a scale when brought to their nearest degrees.

The name or pitch of this scale is next to be discovered. In every tetrachord there are four combinations which occasion a repetition of one of the basses; the name of the scale is always the same as the note repeated.

I must here observe that the inversion of the first diagram has not only discovered a new creation of sounds, but that the synemmenon tetrachord, from whence Guido formed his moll hexachord, is found in the fourth species of the diapason at the second tetrachord, formed by the first flat diesis. [Plate VII.] As this tetrachord had no place in the first diagram, it confirms the necessity of the inversion, without which the mutations in C and B $\flat$  modes would have been imperfect, by not having the auxiliary scales of a fifth lower or higher to move to. The same defect would have precluded this tetrachord from being a  
principal



principal tone of a mode, supported with all its relative scales proper for the mutations. Notwithstanding these great discoveries, we find only two original scales, the one with a minor third, and the other with a major third next the principal fundamental bass or tone; and that this original harmonious constitution of the tetrachord may be moved higher or lower in the mutations, by the use of sharp or flat diesis, in order to support every musical expression proper for every species of voice; so that the solemnity of the grave, the sprightliness of the acute, together with the sweetness of the mean between the two extremes, may move within the compass of their natural powers. For that these constitutions or scales were distinguished by Euclid [p. 2.] as tones, or “a place of the voice capable of system “ without latitude,” cannot be doubted: and when these scales are used successively, according to the order of the mutations, which admits of the increase or decrease of one diesis, the mode is then formed, with its five auxiliary scales added to the principal tone. For as every scale or tone has its principal or governing note, so every mode has its principal or governing scale, to which the other five auxiliary scales refer. And though we are not obliged to use them all, yet it is necessary to know how far their powers extend in the mutations, that a proper keeping with the governing scale may preserve its first impression on the ear. For this purpose one diesis more or less than the original number of the governing scale is the limitation of a simple mode.

From these principles we may form a perfect consonant mutation of a fifth higher or lower, or an imperfect mutation of a third higher or lower than the original pitch of the mode; or

we



we may, from among the different scales, form a dissonant mutation of a tone higher, or lower. These mutations are not difficult, if we attend to a very simple rule, which is, that after the pitch of the governing scale is known, we proceed to form a system of six degrees from that pitch or tone; and if it has a minor third, we must descend from the tone, but if a major third we must ascend in a contrary direction: as for example, in the minor mode of A the system will be A, G, F, E, D, C; but contrary to this in the major mode of C, the system must ascend, C, D, E, F, G, A. These are the names of the scales which may be applied to regulate the mutations in this natural mode. The rule is the same with respect to any number of sharp or flat dieses. As to the distinction between the primary and secondary scales in the major mode, the first, fourth and fifth scales ascending, by having the same species of thirds, are the primary; and the second, third, and sixth ascending, whose thirds are different from the other, are secondary: but in the minor mode, the first, fourth and fifth scales ascending are the primary; and the second, third and sixth descending are the secondary scales.

It will now be proper to turn to Euclid on this very important subject, whose decisive authority will confirm all the foregoing observations.

In page 20, he says, "Mutation is described four ways. By Gender, System, Tone, and Melopæia." He explains each, of these, but rather too concisely for so interesting a subject,

The



The mutation “ by Gender (he says) is when there is a  
“ transition from diatonic to chromatic, or enharmonic, or  
“ from the chromatic or enharmonic to any of the rest.

The number and variety of dieses discovered in the seven  
species of the diapason in each diagram, and applied to the  
genders, are different to illustrate this mutation.

The second “ by System (he tells us) is when the mu-  
“ tation is made from conjunction to disjunction, or the con-  
“ trary.

This mutation by the alteration of one diesis is a fifth lower  
by conjunction, or a fifth higher by disjunction. See the first  
and fourth species of diapason, Plate III. fig. R and S in the first  
diagram, and the contrary in the second, plate VII.

The 3d “ by Tone (he says) is when we make a change  
“ from the dorian to the phrygian, or from the phrygian to  
“ the lydian, or hypermixolydian, or hypodorian; or, in gene-  
“ ral, from any one of the thirteen tones into any of the rest;  
“ and mutations are made beginning from hemitone even unto  
“ diapason.”

If these changes of the tone proceed from the interval of  
diesis to diapason, we have already seen how the system of dia-  
pason, being formed as a scale at any given pitch, is changed,  
and the whole constitution of its parts altered by the introduction  
of a sharp or flat diesis. By this construction, which is a very  
natural



natural one, we come to have a knowledge of the ancient tone or mode, whose natural position can receive no mutation but by a sharp or flat diesis.

With respect to a change from dorian to phrygian, &c. or, in general, from any one of the thirteen tones into any other, the spifs intervals, or genders, sufficiently supply whatever is wanted in compositions the most extensive and elaborate; such as operas, oratorios, and the like; in which a kind of wandering modulation is required to express the recitatives, agreeably to the great variety of sentiment.

These fugitive mutations are transient, serving to diversify and support the several incidents which must arise in long performances, in which the thirteen tones (mentioned by Euclid) may be used with the greatest success.

Euclid, in addition to his former definitions, says [p. 21.]  
 “ Some mutations are made by consonant intervals, others by  
 “ dissonant; and some of these are less concinnous or inconcin-  
 “ nous, some more. Those that have a greater communion  
 “ among themselves are more concinnous, and those that have  
 “ less are more inconcinuous; for in every mutation there must  
 “ be something common, either sound, or interval, or system:  
 “ and this communion is taken according to the similitude of  
 “ sounds; for when similar sounds fall mutually in the muta-  
 “ tions, according as they partake of the spifs the mutation is  
 “ concinnous; but when dissimilar, inconcinuous.”



The “consonant mutation” has already been described by the alteration of a diesis, and the mutation is a fifth higher or lower with the same species of third. Therefore these mutations are more concinnous, by having a more natural communion with each other, than a dissonant mutation, which has nothing common; as for instance, if the change is made from the fifth above the governing scale to the fifth below with the same species of thirds, or the contrary, we have a dissonant mutation, in which the necessary alterations are two dieses; and if one scale is compared with the other, the difference will be a tone major in every part, and consequently not only less concinnous than the other described above, but in every part a dissonant mutation.

With respect to the next part of his definition, wherein he tells us that “in every mutation there must be something common; either sound, interval, or system;” “Sound,” may be the pitch of a scale with a minor third, and afterwards move into another with a major third, with the same name; as from A minor to A major, or from major to minor, in which three dieses make the difference between the two scales.

In this mutation the thirds to the fundamental basses are altered from minor to major, but the fundamental basses are the same; and the whole forms a mixed mode in A, by having two species of thirds. See Plate VIII. In all mutations of this sort there is a difference of three dieses between the two scales.

“Interval”



“Interval” is the next thing which may be common.

The most remarkable interval which distinguishes the character of the scales in their most essential parts is that of the minor or major third; and therefore if the mutation is regulated by either of these intervals, it must be in changing from one minor scale to another, or from a major to a major third.

These mutations, as before observed, will be a fifth higher or lower, which are the perfect consonant mutations; or they may be a tone higher or lower, which have been before distinguished as dissonant.

As to the mutation in which the “Systems” may be common, the seven species of diapason in each diagram are so many examples of this change; for the systems at R are the same as at S, [Plate III.] and yet, by the application of the fundamental basses, we have minor thirds at R, and major thirds at S. The same two species of thirds are also discovered throughout the second diagram at T and U. [Plate VII.] Though the systems are the same in each species of diapason, yet the difference of the pitch of these systems, by the application of the fundamental basses, is a minor third; and the same number of sharp or flat dieses applied to one will be equally necessary to the other. See Plates VIII. and IX.



“ This communion (Euclid says) is taken according to  
“ the similitude of sounds.”

Thus far Euclid has directed the great variety of modulation; and taught us to vary the sentiment by consonant and dissonant mutations, together with a mixed mode, and the imperfect mutation of a minor third higher or lower; but the other imperfect mutation of a major third higher or lower is discovered in Euclid, [page 2.] where he says “ Mutation is a  
“ transposition of a similar into a dissimilar place.”

This description is much more applicable to the mutation of a major third higher than to any of the others; and where the alterations are so much greater they must necessarily be more dissimilar; as for instance, if we change from C major to E minor, the tone or pitch is removed a major third higher, with the addition of one diesis, and a different species of third to each scale.

If we compare this mutation with the consonant one, by disjunction or conjunction, and the imperfect one of a minor third, we discover that in the first it is made by the alteration of one diesis, but the thirds are of the same species; and in the second there is no other difference than in the species of thirds. Hence it appears that the alterations of a diesis, or of the same species of thirds, which in the former mutations acted separately, are now united, and concur to make this a dissimilar mutation. As to the dissonant mutations, they have been distinguished as such by Euclid, and cannot therefore be applied in the present case.

I must



I must observe, that all the mutations are found in the tetrachords, Plates III. and VII. and also Plates VIII. and IX. But so respectable an authority as that of Euclid is not to be neglected, especially as he has explained himself fully on the subject of the mutations, and as his doctrine is conformable to the most established rules of the best modern composers.

Having discovered the laws of the mutations, and the limitations of each mode, which are confined to six scales only, from whence we have an almost endless variety of modulation, which is not only the most animating part of musical composition, but prevents that tiresome monotony of cadence, which is the constant attendant of moving in one scale only ; I shall now consider how the number of modes is fixed and determined by the seven species of diapason in each diagram, agreeably to the Aristoxenian writers ; by which the agreement and harmony of the several parts of this theory will be united and confirmed in a satisfactory manner, and remove every objection that has been made to this part of the ancient harmonica.

There are different opinions among the ancient writers with respect to the number of modes. Aristoxenus, Euclid, and Aristides Quintilianus, mention thirteen. Alypius gives the names and characters of fifteen.

These two opinions are combated by Ptolemy, who has written expressly in favour of seven only ; of whose work the learned Dr. Wallis has given a translation in Latin with notes.



As I do not intend to become a party in the dispute, I shall only assign some probable reason for this difference of opinion, which will be supported by the whole theory, and will lead us to the knowledge of their use and application; but with respect to their names, as they can be of no use to the modern musician, I shall leave them for those who may think them of more consequence.

It has been observed before that Euclid, in describing the seven species of diapason [p. 15.] gives at the same time the names of seven modes, by which, he says, they were called by the ancients, and which I have placed accordingly, in Plate II. fig. 1. But (in page 19.) he tells us that Aristoxenus mentions thirteen tones or modes, and he gives their names, distinguishing some as acute, and others grave.

To reconcile these different opinions will not be difficult, if it be granted (and I think there can be no doubt) that the inversion of the diagram, already explained page 43, was so universally understood as to make a particular description of it unnecessary; and the more so, if we consider that the intervals, tetrachords, hemitones, and tone of disjunction, were the same in each, except the difference of the thirds, and their motion in a contrary direction; and therefore an explanation of the first could be equally applied to the second diagram by those who were acquainted with the doctrine; for the flat dieses were as necessary to perfect this theory as the sharp, and the want of either would have destroyed the whole.

This



This observation is so true, that no practical musician can refuse his assent to it; especially if it be considered that the formation of the spiss intervals and genders in each diagram, which correspond so exactly with the method prescribed by the Aristoxenian writers, could not be discovered on any other principles.

The construction of the several species of diapason, already explained, has proved the necessity of six sharp and six flat dieses. Therefore, if to these twelve dieses we add the natural scale, which is the medium between the two species of sharp and flat dieses, we have exactly the number of thirteen tones or modes mentioned by Euclid.

By this double doctrine of the diagrams (if I may call it so) the ancients are vindicated from the charge of inconsistency, or of giving contradictory accounts of the modes. Some modern writers of reputation have observed that the more ancient of the Greek writers looked upon grave sounds as high, and acute ones as low; and that this distinction was afterwards changed to the contrary, and has since prevailed universally.

If Ptolemy was not acquainted with this double doctrine, it is as easy to account for his limiting them to the seven species of the first diagram, as it is for Alypius to make them fifteen by the addition of two more dieses. It was very easy to fall into this mistake from not observing the limitations prescribed by the diapasons in the two diagrams.

If



If we examine the mutations of the thirteen modes [Plates VIII. and IX.] it will be found that the extreme modes at the sixth sharp and sixth flat cannot be equally perfect with the others, unless these additional dieses are continued, to give a latitude of mutation, by rising a fifth, from a scale of six sharps to one of seven; or to fall a fifth, from a scale of six flats to another of seven.

Yet these auxiliary scales must not be considered as principals, but only as increasing and perfecting the powers of mutation in these extreme modes, in common with the others.

Without attending to this observation, and seeing how much the number of modes depends on those of the dieses, we should determine in favour of the fifteen, mentioned by Alypius, rather than the thirteen of the Aristoxenians, until we recollect the first limitations of the modes, which confined them to the species of the diapason, in which neither of these additional dieses is found; because the notes to which they are applied are the natural and fixed points, from whence the diapasons in each diagram begin their progression. Thus B natural in the first diagram, and F natural in the second, could receive no alteration in their first application; but from the necessity of forming two auxiliary scales to perfect the mutations of six sharps and six flats, the seventh sharp and flat diesis became absolutely necessary; though neither the one nor the other can be a principal note, or pitch of a mode.

These



These extreme sounds are a part of the sesquialter chromatic gender, and form an interval of twenty-one parts of the tetrachord, equal to a deficient fourth peculiar to this gender.

Hence we may conclude that if the spiss intervals within the limits of diapason (as described at Plate IV.) are to determine the number of modes, whose mutations are fully complete, there cannot be more or less than thirteen: for it appears, that if the minor and major scales were each to be formed into modes in separate and distinct classes, and be brought to the account, we should have an even number; because the two natural scales being added to the two belonging to each diesis, would make them continually even numbers. But as no such is contended for among the ancients, we may conclude that the six sharp and six flat dieses, added to the natural mode, complete the whole number, which Euclid and others have mentioned, and which must therefore be limited to thirteen.

I have before taken notice of the Grecian diagrams, which, being a composition of tones major and limmas, have occasioned one of the principal objections by the moderns to their doctrine.

This composition of sounds has always been understood by the moderns to be the scale, or natural degrees of voice, in which the eighths, fifths, and fourths only could be discovered; but as the imperfect concords of thirds and sixes major and minor were not to be found in the diagrams, and as they make so great a part of musical composition, they have con-

M

cluded



cluded it was impossible for the ancients to have any knowledge of harmony, by uniting two or more melodies together.

They did not consider the great advantage of a general arrangement of the seven species of the diapason, the nature and formation of the tetrachords, the genders, and other parts of the harmonica, which would so easily have removed their objections, by supplying them with the imperfect concords, and every other musical interval: nor did they consider the nature and extent of their objection; for if an untuneable, or inconcinuous series of sounds was improper for harmony, it must for the same reason be improper for melody, even if attempted by the most eminent professors.

This last, namely, melody, the advocates for modern improvements in harmony allow the Grecians to have had in very great perfection; but yet deny the possibility of their having any knowledge of combination or harmony, from the very nature and constitution of their scale, or diagram.

As this objection has already been removed by the discovery of the imperfect concords, together with the tone major and minor, semitone major and minor, and many other intervals, which could not be found in a scale of tones major and limmas, but which, according to the rules laid down by Euclid, have been discovered in forming the genders; one may conclude that the scale of tones major and limmas was originally intended for a very different purpose than that to which it has been applied



applied by the moderns. But that I may not draw a conclusion without good and sufficient authority, I shall in support of it shew the original of this scale, its application and importance in a musical theory.

The seven species of diapason, as described at Plate III. fig. R, are regulated agreeably to this scale. In order to understand these properly, we must discover some particular sound, not only the most characteristic in the diapason, but common to each species.

This sound was distinguished by the ancients in every diapason by the name of Mese, and its position was the next sound to the tone of disjunction, which is marked by a cross + in each species of diapason. Plate III.

This mese determines the utmost limits of the tetrachord to which it is applied; and if we proceed to the sound above it, the principal sound, or key, is altered a fifth more acute, which requires an alteration of one diesis to complete the whole.

As the mese was the only sound in every diapason, that required two basses to distinguish and mark the disjunction, its position was regulated by the order in which the seven species of the diapason were placed. By this the place of the principal sound, or key-note, was easily discovered, as well as the number of dieses necessary to form a new scale.



Thus the mese was always found to be a fifth to the principal note of the scale; but when the temperament, or second bass, was applied, it was an eighth to it, and the principal found or key-note of the succeeding tetrachord.

This distinguished character of the mese, supported by the application of the fundamental basses and harmonic principles, prevented the disagreeable effects of three tones in succession; and the sounds that formed the scale are extracted from the harmonics of each tetrachord.

Thus the knowledge of the ancients with respect to harmony appears to have been very great, in not admitting the imperfections of the system of the diapason: a system incapable of harmonic principles without introducing a new scale, or destroying the whole theory by having three tones in succession.

If we examine the seven species of diapason at R, Plate III. we discover that the mese descends in an alphabetical order from the first to the seventh species of diapason thus,  $\widehat{A, G^\sharp, F^\flat, E, D^\flat, C^\sharp, B}$ .

This system, at first sight, has the appearance of being diatonic, but will be found to be tones major and limmas in the Second Part of this work, where the magnitude of all intervals will be determined by the ratio. Thus from A to  $G^\sharp$ , and from E to  $D^\flat$ , will each be found a limma, and the other intervals will be tones major; all of which are formed by the mese, or note of disjunction.

If



If we attend to the regular order in which the seven species of diapason succeed each other, and to the position of the mese, together with the number of dieses necessary to form the several tetrachords, on which the mutations so much depend, we must own this to be a theory deserving our greatest admiration ; not only from the wonderful agreement discovered among the several parts, but as it contains more musical knowledge with respect to harmony, than any theory formed by the moderns.

I shall end this part, which has already been longer than I intended, with one observation, which may put an end to the dispute between the advocates for ancient, and those for modern, harmony ; and which I have never yet met with in any author. This is, that the great, the perfect, and immutable system of the Grecians, so little understood, and almost universally condemned by the moderns, is a very essential part of our own theory : for if the principal sounds, or pitch, of the several modern scales are placed in an alphabetical order, in the same manner as the mese, the relation of each principal sound to the next will be either a tone major or limma.

In consequence of this observation it appears that the ancients had as good a claim to the knowledge of harmony as the moderns. The truth is, the principles of science have always been the same, and are governed by the same fixed and unerring laws, though they are not always equally understood : and I wish to have any concord, or other musical interval, pointed out, which is not to be found in the harmonica of the Grecians.

As



As I am desirous of removing every objection to the Grecian doctrine, I must not pass over a very material one."

Aristoxenus says [p. 20.] "that many intervals are modulated less than diatessaron, but are all dissonant."

If this were true, all systems must be discords, which would destroy the harmonious constitution of the tetrachord; but as he cannot be understood to have any such intention, it must relate to the movement of the roots or fundamental basses, which move by fifths, or fourths the inversion of the fifths: this will be easily understood; for in this theory we have no instance of the roots moving a tone, and the third is the only motion that remains; the consequence of which will be a change of key from major to minor, or the contrary, and will occasion a temperament that always attends a fourth term; which, in the Second Part, in treating of discords, will be more particularly explained. At present it is sufficient to observe, that no such motion of falling or rising a third can be found among the fundamental basses of the tetrachords, except at the disjunction at S, Plate III. which changes the scale a fifth higher, and is not one of the primes, but is a secondary fundamental bass, or fourth term, and moves immediately into one of the primes of the next tetrachord.

This explanation discovers that this part of Aristoxenus's doctrine must be applied to the roots, and not to the systems or melodies.

Euclid



Euclid [p. 8.] mentions something similar to what Aristoxenus has observed. He first enumerates the species of consonances, and then says that “all intervals less than diatessaron are dissonant,” and tells us what they are; such as “diesis, hemitone, tone, trihemitone, and ditone.”

In this account of dissonant intervals there is no mention made of the limma; but this very account furnishes a proof of Euclid's being well acquainted with the imperfect consonances; notwithstanding they are excluded from a place in the diagrams, in the same manner as the limma is in all other places of the harmonics.

As the same objection to all intervals less than diatessaron being admitted among the primes, or fundamental basses, must remain, as before mentioned, we may observe that the diagrams are not disordered by introducing the imperfect consonances, but are preserved in their first and original state of tones major and limmas, as fundamental basses.

It may be expected that the discords which make so essential a part of a musical theory should be treated of next, but as they will be better understood when explained by the Ratio, I shall reserve them, together with their preparations and resolutions, for the Second Part.

These observations do not so properly come under the sixth general head of the harmonica, according to Euclid; yet as the knowledge of the primary fundamental basses is so essential  
to



to the mutations, it was necessary to place them in that light which their importance demands.

I now proceed to the Melopæia, or Seventh and last Part of the Ancient Harmonica.

## VII. Of the MELOPÆIA.

The Melopæia is that part of the harmonica which determines the motion, or succession, of sounds proper to produce air, or melody; and, according to Euclid [p. 22.] consists of Ductus, Nexus, Petteia, and Extensio.

“Ductus” is a gradual succession, either ascending or descending.

“Nexus” directs the succession to be by harmonic intervals, such as a 3d, 4th, 5th, or 8th, and the like.

“Petteia” is a repetition of a sound in the same tension.

“Extensio” is a continued singing of a sound in the same tension.

Aristides Quintilianus [p. 29.] has given many divisions and subdivisions of the Melopæia; but if it means no more than simply to determine all possible motion of sounds, without  
any



any regard to the various combinations of time, the four parts mentioned here by Euclid give every thing that the subject demands with respect to a simple melody ; which, notwithstanding, may be applied to two or more, with equal success, in the following order :

“ Ductus,” or a succession of degrees :

Two, and sometimes three, melodies may move in similar motion ; and two in the contrary, either ascending or descending.

Four melodies may move in succession by uniting similar and contrary motion.

“ Nexus,” or the motion of harmonic intervals :

Two, and sometimes three, melodies may move by harmonic intervals in the similar motion ; or three, and sometimes four, in the contrary.

The two motions of the Ductus and Nexus may be variously changed, and applied to one, two, or more melodies.

“ Petteia,” or the repetition of a sound in the same tension :

By this the various combinations of time may be described, but nothing relating to tune : or it may be applied as a basis to every sound in succession in the melodies ; for in this last

N

instance



instance, as the number of pulsations, or repetitions to divide the time, is not limited, such an application of the Petteia to sounds in succession will constitute air, or melody, and when united with the other parts of the Melopæia, in two or more melodies, is capable of producing great and noble effects.

“ Extensio :”

To know how to apply this part of the Melopæia, it will be necessary to discover if a sound, continuing in the same tension, can be common to two or more fundamental basses.

Of this we have an example in all basses moving a fifth; as in the disjunction of the tetrachord at R, Plate III. where the last note of the tetrachord is a fifth to the first bass, and an eighth to the second, and continues in the same tension during the change of the bass and two of the harmonics.

All ligatures, syncopation, and the preparation of discords, are so many applications of extension to two basses, and are in no respect contrary to the laws of harmonics. But if we consider extension in its greatest latitude, it will be found to be an application proper for three, four, or any number of basses or melodies in succession: for as extension, when confined to a simple sound, has not in itself any of those varieties we meet with in tune and time, it follows, that the motion of other sounds placed in opposition to it, according to the three first parts of the Melopæia before described, must procure us this effect. And if any discords should be generated by the sounds in motion, when compared with the note of extension, they



they are rather accidental than essential, because they require no other resolution than that of continuing the motion of those accompaniments, till they finally terminate in a consonant proportion.

Thus extension being applied to C, and the bass moving C, D, E, or E, D, C, at the same time, we have in this case the discord of the seventh at D, the second sound, which is resolved by the motion of the third sound in the bass at E, without any alteration in the extension; and it would be the same if the thirds to the basses and other harmonic sounds were added. But if any discords are generated among the melodies in motion, as frequently happens, they must be regularly prepared and resolved, according to the rules of discords, but without interfering with the extension, which may be continued any quantity of time that the composer pleases.

Many examples might be brought from the best writers, if what has been already said on this subject were not sufficient to explain this important part of the Melopæia; without which harmony would lose a great part of its beauty, and be robbed of its principal and most noble effects.

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I have now gone through the Seven Parts into which the Ancients divided the Harmonica. I have given each part a musical construction, by which their mutual relation and agreement have been explained, and reconciled to the laws of harmony, beginning with the first idea or definition of a musical sound, and proceeding to the knowledge of intervals, or the distance to be observed between one sound and another in  
the



the formation of genders, systems, tones, or modes; from which we have been able to form and regulate the several scales, and discover the relation which every sound in a scale has to its principal or fundamental sound; likewise how any given scale, taken as a principal one, is connected with others, by which, under certain limitations with respect to the dieses or position of the hemitones, the consonant and dissonant mutations are regulated; the number also of sharp and flat dieses necessary to perfect every scale, has been collected and disposed in a particular order, proper for the discovery of the genders, as well as the fifths and all other intervals, which can be wanted in the most elaborate compositions.

I was induced to, and encouraged in, this extensive and arduous task by some discoveries which appeared to be of the greatest importance towards the undertaking and explaining of a theory, which had for many ages been only a subject of dispute; neither party being able to determine any thing conclusive in support of their different opinions, either for, or against, the harmonic principles of the Grecian doctrine. Nor could I have flattered myself with better success, had not the inversion of the first diagram offered something the most interesting and agreeable to my wishes. To this succeeded the order of placing the seven species of diapason in each diagram, which encouraged me yet more to proceed; but when the tetrachords in their various positions could not be formed without the sharp and flat dieses, and the conjunction and disjunction could not be explained without the application of the harmonic principles, I remained no longer in doubt, but was fully convinced that without a perfect knowledge of harmony it must have been  
been



been impossible to have formed a theory so expressive and curious as the Grecian in all its parts; nor can it be understood, unless explained by the same laws, by which it was originally formed.

However, that I may give some satisfaction to those who object to all theories not demonstrated by numbers and supported by the Ratio, I have determined to try how far the power of numbers will carry me in a theory of harmonics, agreeably to the *Pythagorean* doctrine, which will be the subject of the Second Part of this Work.

END OF THE FIRST PART.

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INTRO-